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Vibration Analysis Techniques for Faults Detection of Multi-cracked Rotor System

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Abstract—*Rotor is important element in a variety of industrial applications. An unexpected failure of the rotor system may cause significant economic losses and accident. For this reason, fault detection in the rotor system has been the subject of intensive research. Vibration signal analysis has been widely used in the fault detection of rotating machinery. This paper develops the finite element model of the rotating shaft with multi-cracks. The analytical method for the calculation of the natural frequencies of such a rotor system is investigated and the modeling of the open cracks element is discussed. The natural frequency of the experiment system is measured for various cases of positions and depths of the cracks by using Picoscope. By comparing both the theoretical and experiment results of the natural frequencies, the accuracy of the developed FEM of the rotating shaft with multi-cracks is clarified.*

1. INTRODUCTION

All metal members that are subjected to vibration and cyclic stresses in more or less localized areas, cracks may occur. Since cracks cannot be easily seen with the naked eyes, the non-destructive testing methods like ultrasonic testing, X-ray, etc. can be used to detect them. However, these methods are costly and time-consuming for complex or large structures. For this reason, the vibration-based structural health monitoring methods, especially those based on the change of modal parameters (frequencies, shape and damping), have been explored for detecting cracks. These vibration-based techniques have been applied to a variety of engineering structures, such as beams, trusses, rotors, etc.

In the past most of the research work has been done on a structure with a single transverse surface crack. When more than one crack appears in a structure, the dynamic response becomes more complex depending upon the relative positions and depths of these cracks. For the first time, Dimaragonas et al. [1] suggested an analytical method for the computation of dynamic response of a cracked Euler-Bernoulli beam by modeling the cracked region as a local flexibility resulted from fracture mechanics. Christides et al. [2] developed a continuous theory for vibration of a uniform Euler-Bernoulli beam containing one or more pairs of symmetric cracks. A differential equation of motion and corresponding boundary conditions are given in this paper using the Hu-Washizu variational principle. Darpe et al. [3] detected that various combinations of position and depth can lead to the identical changes in the natural frequencies. Ostachowicz et al. [4] analysed the effect of positions and depths of two cracks on the natural frequency of cantilever beams. Shen et al. [5] have analysed a pair of symmetric cracks at mid-span and focused their attention on the effect of these cracks on the mode shapes. Ruotolo et al. [6] studied the effect of crack depth and location on the eigenfrequencies of a double cracked beam. Al-said [7] developed a mathematical model describing the lateral vibration of a stepped cracked beam carrying concentrated masses and obtained a global effect of cracks to the system. The advantage of the proposed algorithm is to identify the crack by monitoring a single natural frequency of the system. Ranjan et al. [8] studied experimentally the variation in vibration characteristics of multi cracked rotating shaft using piezoelectric sensor. And they observed from the numerical results that, there were appreciable changes in vibration characteristics of the rotating shaft with and without cracks which can be utilized for multi cracks identification of structures. Sekhar [9] carried out a parametric study of two transverse open cracks in a rotor and studied the effect of various crack parameters on the eigenfrequencies and stability speeds of rotors. He used finite element model of a rotor bearing system for flexural vibrations and carried out a study on two aligned open cracks. Dong et al. [10] introduced finite element (FE) model, which is based on a transfer matrix analysis and local flexibility theorem to obtained crack identification of a static (non-rotating) rotor with an open crack. Han et al. [11] the continuous wavelet transform of the measured wave signals was used to detect the damage location. In this method the magnetostrictive effect was employed for a non-contact measurement of stress waves in rotating shafts. Masoud et al. [12] suggest a mathematical model to study the effect of crack depth on the transverse vibration characteristics of a pre-stressed -fixed cracked beam. They studied the effect of interaction between the crack depth, and axial load on the beam natural frequencies. An experimental verification was carried out for the obtained theoretical results. Chati et al. [13] studied the dynamic characteristics of a cantilever beam having a transverse edge crack by using modal analysis. In the field of non destructive evaluation (NDE), neural networks are a very useful tool for analyzing and filtering different variety of measured quantities and signals. Kang et al. [14] used a neural network approach to determining fatigue crack configuration. Etemad et al. [15] proposed indirect method of diagnosing a shaft using neural networks. They obtained natural frequencies by means of a finite element method. Then those Numerical data were used to train three two-layer feed-forward back-propagation neural networks. He Y. et al. [16] proposed a genetic algorithm based method for shaft crack detection. Dharmaraju et al. [17] developed a general identification algorithm to estimate crack flexibility coefficients and crack depth based on the beam force–response information. They used an Euler–Bernoulli beam element in the finite element modeling, and the crack has been modeled by a local compliance matrix, which has four degrees of freedom. Zheng et al. [18] applied a finite element method to obtain the natural frequencies and mode shapes of a cracked beam. They obtained the flexibility matrix for cracked beam by adding the crack flexibility to the flexibility matrix of the intact beam element as an overall additional flexibility matrix instead of adding it as local flexibility matrix; using this derivation, they were able to predict the natural frequencies more accurately.

In this paper, an applicable approach is proposed to estimate specification of rotor shaft cracks using theoretical analysis and experimental analysis. The approach evolved in this paper intimates location and depth of the open cracks in the rotor. The comparison results in both methodologies are written above are performed. The results of theoretical analysis and experimental analysis are compared. The test results show that the proposed FEM model is able to estimate the crack specifications with high accuracy.

2. MATHEMATICAL MODELING

It is assumed that the crack changes only the stiffness of the structure whereas the mass and damping coefficients remains unchanged. Cracks occurring in structures are responsible for local stiffness variations, which in consequence affect the mode shapes of the system. Following Fig. shows the cross-section of the cracked shaft.

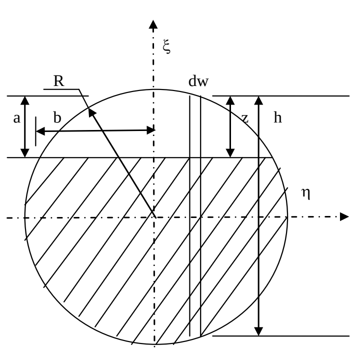

Fig. 1: The cross-section of the cracked shaft

According to the fracture mechanics and the energy principle of Paris, the additional strain energy due to a crack is given by the following equations:

$$U=\int J(A)dA \tag{1}$$

where J (A) is strain energy density function with only bending deformation taken into consideration. It is expressed as:

$$J(A)= 1/E' K_I^2 \tag{2}$$

where E'= E /(1‒ν) , ν is the Poisson ratio and K_I is the stress intensity factors corresponding to the bending moment M in plane crossing the axis of the beam.

The local flexibility due to the crack in the ξ -axis direction

$$C_\xi=\frac{\partial^2 U}{\partial M^2}=\frac{1-v^2}{E}\int_{-b}^{b}\partial\eta\int_0^b\frac{32}{\pi^2 R}(R^2-\eta^2)\pi z F_2^2(z/h)dz \tag{3}$$

The similar expression of local flexibility in the η -axis direction can be written as

$$C_\eta=\frac{\partial^2 U}{\partial M^2}=\frac{1-v^2}{E}\int_{-b}^{b}\partial\eta\int_0^b\frac{32}{\pi^2 R}\eta^2\pi z F_1^2(z/h)dz \tag{4}$$

Now, we can get the local flexibility matrix C_1 of the element with a crack

$$\boldsymbol{C_1}= \begin{bmatrix} C_\xi & 0 \\ 0 & C_\eta \end{bmatrix} \tag{5}$$

Here, the coupled flexibility is neglected since it is much less than the element in the main diagonal of the C_1. Let C_0 is the flexibility matrix of uncracked shaft element.

The total flexibility matrix C is expressed as:

$$C=C_0+C_1 \tag{6}$$

Thus, the total stiffness matrix of the crack element is written as

$$Kc =TCT^T \tag{7}$$

where T is the transformation matrix and given by

$$T=\begin{bmatrix} -1 & -l & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{8}$$

Assembling all the element mass, damping and stiffness matrices of the rotor system in stationary coordinate system, the equation of motion in stationary coordinate is

$$M\ddot{z} + C\dot{z} + Kz = F \qquad (9)$$

where, M, C, K and F are total mass, damping, stiffness and external exciting force matrices of rotor system respectively. Z is the displacement of the element node.

For the rotor system considered here, each beam element has two nodes and each node has two degrees of freedom representing transverse and deflecting displacements in the corresponding cross-section. Here, only the mode shape in the ξ-axis direction is discussed by assuming the rotor system is rigid supported at the bearing position.

The mode shape can be obtained by solving the homogeneous part of (9) without considering the effect of the damping.

$$M\ddot{z} + Kz = 0 \qquad (10)$$

Substituting $z_i = A^i sin(\omega_i t + \varphi_i)$ we get

$$(-\omega_i^2 M + K)A^i = 0 \qquad (11)$$

Where ω_i and A^i is the i-th nature frequency and eigenvector (mode shape).

3. DATA ACQUISITION

The shaft studied in this paper was a homogeneous shaft with perfectly round surface sections and had a length of 20.0cm and a diameter of 2.5cm. Regarding the shaft specifications, mild steel, with a density of 7800 kg/m^3 was considered. The Young's modulus for this shaft was 2.1x1011N/m^{2}and the Poisson's ratio was 0.3. Shaft was rotating at a speed of 1400rpm.

ANSYS is very powerful finite element software capable of providing natural frequencies of a shaft with given crack's specifications including location, depth, and width. Center of the shaft is chosen as the reference point of the shaft and all the crack locations are specified with respect to this point. Fig. 5 shows the first modal frequency of the cracked shaft in ANSYS. The maximum selected crack width is 0.2mm which is considered a severe crack. Overall, 7 sets of numerical input-output data of different cracks are obtained. Table 1 presents 7 samples of the obtained numerical data.

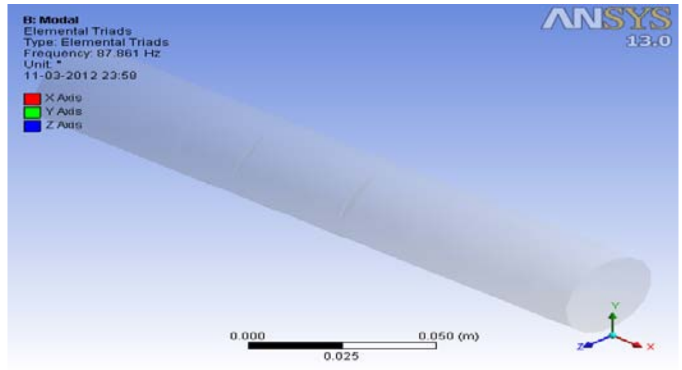

Fig. 2: ANSYS model

Table 1: Theoretical Data Obtained Using Finite Element Method

Crack No.	Depth of crack (in mm)	First Crack Location (in mm)	Second Crack Location (in mm)	ω in Hz
1	1.0	-10	10	70.74
2	2.0	-10	20	57.47
3	2.5	-15	25	57.70
4	3.5	-20	30	57.38
5	4.0	-25	35	58.86
6	4.5	-35	45	57.49
7	5.0	-30	40	53.43

For the experiment the cracks were assumed to be perpendicular to the shaft main axis. Cracks were done by wire electrical discharge machining (WEDM) using Ø 0.25mm brass wire. For this type of cracks, two important properties were defined: locations and depth. It is also necessary that the cracks be tangent to a vertical plate. Usually, a shaft was connected to a motor through a flexible coupling, and it was supported by radial bearings at both ends. It was possible to calculate the natural frequencies of a shaft using analysis of the data gathered by some vibration sensors such as piezoelectric sensors connected to the set of bearings. Fig. 3 shows a perpendicular crack on a shaft installed with a piezoelectric sensor.

Fig. 3: Photograph of shaft installed with a piezoelectric sensor

A piezoelectric sensor generates an electric signal when the outer surface of the sensor is compressed. Such a mechanical input-electrical output sensor is used in many applications to detect the vibration signals and extract the natural frequencies. After the vibration signal has been obtained by the Piezoelectric Sensor, it will be gathered by a data acquisition system for signal processing. The vibration signal in the time domain will be transferred into the frequency spectrum by applying the Fast Fourier Transform (FFT). Then the power spectrum – a measurement of the power at various frequencies – will be obtained by multiplying the FFT results by their conjugates. Fig. 3 and Fig. 4 show the power spectrum for the vibration signal. It is interesting to note that frequency at the peak decreases with cracks. In case of no crack the frequency is 135Hz and in case of double it is 74.74Hz.

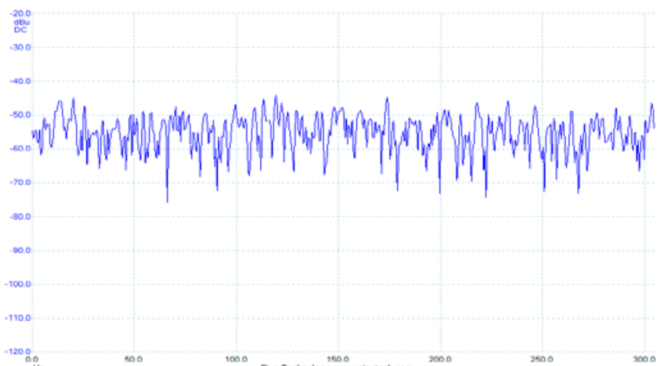

Fig. 4: Frequency spectrum of shaft without-crack

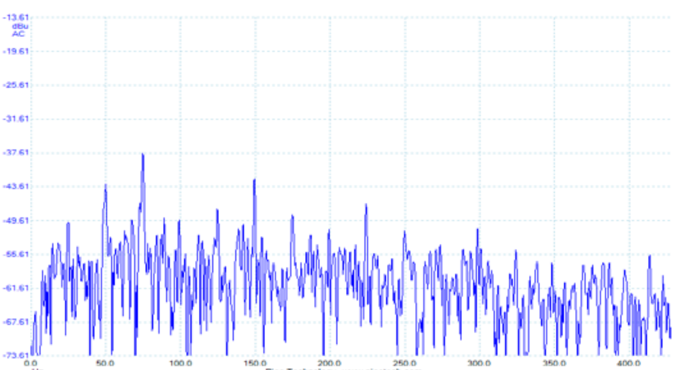

Fig. 5: Frequency spectrum of shaft with multi-cracks

Table 2: Experimental Data Obtained Using PicoScope

Crack No.	Depth of crack (in mm)	First Crack Location (in mm)	Second Crack Location (in mm)	ω in Hz
1	1.0	-10	10	74.74
2	2.0	-10	20	59.70
3	2.5	-15	25	60.72
4	3.5	-20	30	60.38
5	4.0	-25	35	61.86
6	4.5	-35	45	59.49
7	5.0	-30	40	55.43

4. RESULT AND DISCUSSIONS

The results from theoretical analysis and experimental analysis are shown in the Table 1 and Table 2. It can be notice that the variation in the frequency is more visible with depth of the cracks. The crack depth is a measure to diagnose the severeness of the crack. Therefore, the mentioned precision regarding the crack depth estimation is of great importance. For a shaft with deeper crack(s) more care should be provided to prevent damage and total break of the shaft.

5. CONCLUSION

In this paper an indirect non-destructive approach is proposed for rotor cracks detection. A finite element model is used for flexural vibration analysis of a rotor with two open cracks and its experiment verification has been investigated, and the following points were clarified:-

(i) The concise, accurate and general-purpose oriented model of the open cracks in a rotor system has been developed.

(ii) The analytical calculation method of the natural frequency of rotor system with open cracks has been deduced.

(iii) The natural frequency has been calculated for various cases of position and depth of cracks by using the developed open cracks model, and it has been confirmed experimentally. The analysis and experiment clarify the validation of the developed open cracks models.

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